

Time-domain Finite Element Method for Transient Electric Field and Transient Charge Density on Dielectric Interface

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Abstract—This paper is devoted to solving the transient electric field and transient charge density on the dielectric interface under the electroquasistatic (EQS) field conditions with high accuracy. The proposed method is suitable for both 2-D and 3-D applications. Firstly, the governing equations represented by scalar electric potential are discretized by the nodal finite element method (FEM) in space and the finite difference method in time. Secondly, the transient constrained electric field equation on the boundary (TCEFEB) is derived to calculate the normal component of the transient electric field intensities on the Dirichlet boundary and dielectric interface as well as the transient charge density on the dielectric interface. Finally, a 2-D numerical example is employed to demonstrate the validity of the proposed method. Furthermore, the comparisons of the numerical accuracy of the proposed method in this paper with the existing FEMs for electric field intensity and charge density on the dielectric interface are conducted. The results show that the numerical accuracy of the proposed method for calculating the normal component of transient electric field intensities on the Dirichlet boundary and dielectric interface as well as the transient charge density on the dielectric interface is close to that of nodal electric potential and an order of magnitude higher than those of existing FEMs.

Index Terms—Electroquasistatic field, composite insulation structure, transient constrained electric field equation on the boundary, finite element method, interface charge, time-domain.

I. INTRODUCTION

WITH the rapid development of renewable energy and the application of DC transmission technology, various types of power electronic apparatus with high voltage and large capacity are being widely used in the transmission grid. The working voltages applied to the components of these apparatus are no longer just AC voltage or DC voltage, but more complex voltage waveforms, such as composite voltages with AC and DC [1], unipolar periodic pulse voltage [2], and so on. In

order to improve the insulation strength of these apparatus, composite insulation systems with multiple dielectric materials are employed [3], [4]. In general, the maximum electric field intensity in a composite insulation system often occurs on the surface of the conductor or the dielectric interface [5]. Under the above complex waveforms of voltage, it is necessary to consider both displacement current and conduction current when analyzing the distribution of the transient electric field. Charges will accumulate on the dielectric interface due to the discontinuity of the permittivity. The time-varied accumulated charges will influence the distribution of transient electric field intensity in the combined insulation structures [6]. As a result, it is of great academic value and application significance to accurately calculate transient electric field intensity on the conductor surface and on the dielectric interface as well as the transient interface charge density for optimal design of composite insulation structures.

In general, the calculation of the transient electric field under the above circumstance should follow the electroquasistatic (EQS) field equations in which the electric field induced by the varied magnetic field is ignored [7]. The EQS approximation is effective when the characteristic length of the electromagnetic system is much smaller than the distance that the electromagnetic wave travels during the characteristic time of the electromagnetic system [8]. Therefore, the EQS approximation is widely used in the calculation of the transient electric field of high-voltage equipment [9]–[11]. To numerically calculate the electric field intensity based on the EQS equations, spatial discretization and time derivative should be conducted. Generally, there are two spatial discretization methods for governing equations of the EQS field, the finite integration technique (FIT) [12], [13], and the finite element method (FEM) [5], [9]. While for the time derivative, it can be solved by the direct integration method [6], the modal method [14], and the state-space method [15]. Moreover, some new unconventional methods are also reported to solve the partial differential equations, such as isogeometric analysis [16], mesh-free methods based on deep neural networks [17], and so on. At present, the time-domain FEM is commonly employed, in which the spatial discretization is realized by FEM, and the time discretization is achieved by the direct integration method [18]–[20].

Due to the continuity of the electric potential on the dielectric interface for the composite insulation structure, the nodal FEM is suitable for solving potential functions with

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high numerical accuracy. The electric field intensities on the conductor surface and the dielectric interface are commonly obtained by calculating the gradient of the nodal potential function. However, if the simplex element is used in nodal FEM, the obtained electric field intensity is regarded as the average electric field intensity at the element. Then, the nodal electric field intensity can be calculated by taking a weighted average of the obtained elemental electric field intensity [21]. The numerical accuracy in the calculation of the nodal electric field intensity is much lower than that of the nodal potential function because of the numerical differential. Due to the discontinuity of the normal component of the electric field intensity on the dielectric interface, it is difficult to accurately calculate the normal component of the electric field intensity on the interface, and the accuracy of the interface charge density is even lower.

At present, there are three types of methods for improving the accuracy of electric field intensity in the field domain. The first one is to improve the accuracy of nodal potentials by increasing the mesh density and using high-order interpolation functions [22]. Due to the existence of a differential error, the accuracy of the electric field intensity can only be slightly improved. The second one is to solve the electric field intensity in the local area around the node, including potential theory [23] and super-convergence theory [24]–[27]. The method of potential theory also needs to consider the divergent of the potential function, thus the accuracy of the electric field intensity is not greatly improved. The super-convergence theory employs the interpolation of the electric field intensity of the Gauss point with the highest precision within the element to obtain nodal electric field intensity, which can effectively improve the accuracy. The third one is to solve the nodal electric field intensity in the whole field domain, including using of complementary variational principles [28] and representing the electric field intensity with 3-D facial vector basis functions [29]. This method avoids calculating the gradient and can efficiently improve the accuracy of electric field intensity. The above mentioned three methods are all aimed at improving the accuracy of the nodal electric field intensity within the field domain, but the nodal electric field intensities on the boundary and dielectric interface are still not being handled well.

For more accurately calculating the electric field intensities on the boundary and dielectric interface, the edge FEM was used [30]–[32]. The edge element selects the field vector as the independent variable to ensure that the electric field intensity is continuous in the tangential direction without forcing it to be continuous in the normal direction. However, the high calculation cost is unavoidable in edge FEM because the field vector must be calculated. In addition, a hybrid method of the FEM and boundary element method (BEM) with high accuracy for electric field intensity was proposed [33], [34], but one more step of calculating electric field intensity on the boundary by BEM was essential compared with FEM. For calculating the electric field intensity on the Dirichlet boundary, the constrained electric field equation on the boundary was proposed, which can directly and accurately calculate the normal component of the electric field intensity on the

Dirichlet boundary [35]. The above methods are implemented in the electrostatic field, and cannot be directly applied in the EQS field condition.

In this paper, a time-domain FEM is proposed to calculate the normal component of the transient electric field intensities on the Dirichlet boundary and the dielectric interface as well as the transient interface charge density under the EQS field. This paper is organized as follows. In Section II, the initial value and boundary value problem is introduced, and the time-domain finite element equation is obtained after discretizing in speciality and time. In Section III, the transient constrained electric field equation on the boundary (TCEFEB) is derived, and the calculation method for the transient charge density on the interface is proposed. To testify the validity of the proposed method, a 2-D numerical example is presented in Section IV. Finally, conclusions are presented made in Section V.

II. GOVERNING EQUATION AND DISCRETIZATION IN THE EQS FIELD

Under the condition of EQS approximation, the magnetic induction of Maxwell's equation can be neglected. As a result, the electric field is approximately free of rotation, and the scalar electric potential function φ can be introduced as $\mathbf{E} = -\nabla\varphi$, where $\mathbf{E}$ is the electric field intensity. Then, the governing equation of the scalar potential in the EQS field can be obtained,

$$\nabla \cdot \left[\gamma \nabla \varphi + \frac{\partial}{\partial t} (\varepsilon \nabla \varphi) \right] = 0 \quad (1)$$

where ε and γ are permittivity and conductivity of the medium, respectively, and t is time.

The governing equation of scalar potential in (1) can be mathematically classified as a transient diffusion problem and belongs to the parabolic partial differential equation [36].

A. Initial Value and Boundary Value Problem in the EQS Field

As depicted in Fig. 1, let field domain $\Omega \subset \mathbb{R}^2$ be a convex polygonal domain, the interface Γ_{12} separates Ω into two subdomains with different medium Ω_1 and Ω_2 , thus $\Omega = \Omega_1 \cup \Omega_2 \cup \Gamma_{12}$. Let $\partial\Omega$ be the boundary of domain Ω , Γ_1 is the Dirichlet boundary where the electric potential is given, Γ_2 is the Neumann boundary where the normal component of electric potential is given, such that $\partial\Omega = \Gamma_1 \cup \Gamma_2$.

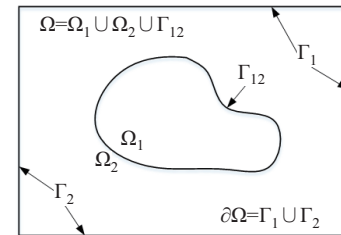


Fig. 1. A sketch of the domain and boundary for the initial value and boundary value problem.

In the EQS field, the initial value and boundary value

problem for scalar electric potential can be expressed as:

$$\begin{cases} \nabla \cdot [\gamma \nabla \varphi + \frac{\partial}{\partial t}(\varepsilon \nabla \varphi)] = 0 \\ \varphi|_{\Gamma_1} = u(t) \\ \frac{\partial \varphi}{\partial n}|_{\Gamma_2} = \psi(t) \\ \varphi|_{t=0} = \varphi(0) \end{cases} \quad (2)$$

where $u(t)$ and $\psi(t)$ are given functions corresponding to the Dirichlet and Newmann boundaries, respectively, and $\varphi(0)$ is the initial distribution of the electric potential.

The corresponding EQS condition represented in the form of the electric potential on the interface of the mediums are:

$$\varphi_2 - \varphi_1 = 0 \quad (3)$$

$$\gamma_2 \frac{\partial \varphi_2}{\partial n} - \gamma_1 \frac{\partial \varphi_1}{\partial n} = -\frac{\partial}{\partial t} \left(\varepsilon_2 \frac{\partial \varphi_2}{\partial n} - \varepsilon_1 \frac{\partial \varphi_1}{\partial n} \right) \quad (4)$$

where the subscript of 1 and 2 represent the corresponding variable and basic parameters in the different medium Ω_1 and Ω_2 respectively, $\mathbf{n}$ is the unit normal vector on the dielectric interface Γ_{12} , and the direction of $\mathbf{n}$ is from medium 1 to medium 2.

Charges will accumulate on the interface of Γ_{12} because of the discontinuous of the permittivity and the conductivity in different medium Ω_1 and Ω_2 . Interface charge density σ can be expressed as:

$$\sigma = \varepsilon_2 E_{2n} - \varepsilon_1 E_{1n} \quad (5)$$

where E_{1n} and E_{2n} represent the normal component of the electric field intensity on the interface of the medium 1 side and medium 2 side, respectively, and $E_{1n} = -\partial \varphi_1 / \partial n$, $E_{2n} = -\partial \varphi_2 / \partial n$.

The calculation of the transient electric field intensity and the transient charge density on the interface with high accuracy is the key point in this paper.

B. Discretization of Governing Equations under the EQS Field

In order to solve the initial value and boundary value problem in (2) to (4), the time-domain FEM is employed.

The field domain Ω was discretized into m finite elements with n nodes. Among m elements, m_1 elements are collinear with the Dirichlet boundary Γ_1 , and m_2 elements are collinear with the Newmann boundary Γ_2 . In addition, there are m_3 elements in Ω_1 which are collinear with the dielectric interface Γ_{12} , similarly, there are m_3 elements in Ω_2 which are collinear with the dielectric interface Γ_{12} . Assume there are $(n-n_1)$ nodes on the Dirichlet boundary Γ_1 . When numbering the nodes, the nodal potentials of the nodes that are in Ω but not on the Γ_1 are numbered first and expressed as $\varphi_1, \varphi_2, \dots, \varphi_{n-1}$. Then, the nodes on the Γ_1 are numbered in sequence and expressed as $\varphi_{n+1}, \varphi_{n+2}, \dots, \varphi_n$.

For node i of element Ω_e with a boundary of element $\partial\Omega_e$, multiplying the governing equation of (2) with a nodal basis function N_i^e and integrates over Ω_e we can derive:

$$\int_{\Omega_e} N_i^e \nabla \cdot \left[\gamma \nabla \varphi + \frac{\partial}{\partial t}(\varepsilon \nabla \varphi) \right] d\Omega = 0 \quad (6)$$

where N_i^e is the nodal basis function at node i .

Using the Green's formula, the integral of (6) can be decomposed into two parts, the integral for the field domain

and the boundary, respectively.

$$\begin{aligned} & \int_{\Omega_e} \nabla N_i^e \cdot \left[\gamma \nabla \varphi + \frac{\partial}{\partial t}(\varepsilon \nabla \varphi) \right] d\Omega \\ &= \oint_{\partial\Omega_e} N_i^e \left[\gamma \frac{\partial \varphi}{\partial n_e} + \frac{\partial}{\partial t} \left(\varepsilon \frac{\partial \varphi}{\partial n_e} \right) \right] d\Gamma \end{aligned} \quad (7)$$

where, $\mathbf{n}_e$ is the unit normal vector of the boundary $\partial\Omega_e$, and the direction of $\mathbf{n}_e$ is the outer normal direction of $\partial\Omega_e$.

The basis function for linear interpolation is used to discretize the potential function in each element. Thus, the approximate function for electric potential can be given as:

$$\tilde{\varphi}^e = \sum_{j=1}^{n_e} N_j^e \varphi_j \quad (8)$$

where φ_j is nodal potential at node j , N_j^e is nodal basis function at node j with $N_j^e(x_i) = 0$ when $i = j$ and 0 otherwise, and n_e is the total number of nodes at Ω_e .

Inputting (8) into (7), the finite element equation at Ω_e can be expressed as:

$$\begin{aligned} & \sum_{j=1}^{n_e} \int_{\Omega_e} \nabla N_i^e \cdot \left[\gamma \nabla N_j^e \varphi_j + \frac{\partial}{\partial t}(\varepsilon \nabla N_j^e \varphi_j) \right] d\Omega \\ &= \oint_{\partial\Omega_e} N_i^e \left[\gamma \frac{\partial \varphi}{\partial n_e} + \frac{\partial}{\partial t} \left(\varepsilon \frac{\partial \varphi}{\partial n_e} \right) \right] d\Gamma \end{aligned} \quad (9)$$

Adding (9) together with all the elements, we can form finite element equations in the whole field of domain Ω ,

$$\begin{aligned} & \sum_{j=1}^n \left\{ \sum_{e=1}^m \int_{\Omega_e} \nabla N_i^e \cdot \left[\gamma \nabla N_j^e \varphi_j + \frac{\partial}{\partial t}(\varepsilon \nabla N_j^e \varphi_j) \right] d\Omega \right\} = \\ & \sum_{e=1}^m \oint_{\Gamma_e} N_i^e \left[\gamma \frac{\partial \varphi}{\partial n_e} + \frac{\partial}{\partial t} \left(\varepsilon \frac{\partial \varphi}{\partial n_e} \right) \right] d\Gamma \quad (i = 1, 2, \dots, n) \end{aligned} \quad (10)$$

Considering physical meaning, $-\partial \varphi / \partial n$ stands for normal component of the electric field intensity E_n . Thus, the closed curve integral along the boundary in the right hand of (10) can be divided into three parts. The first part is the curve integral along with the Dirichlet boundary where $\partial \varphi / \partial n$ can be represented as $-E_n$. The second part is the curve integral along with the Newmann boundary where $\partial \varphi / \partial n$ can be represented as ψ . And the third part is the curve integral along with the dielectric interface on both sides. Thus, (10) can be written as:

$$\begin{aligned} & \sum_{j=1}^n \left\{ \sum_{e=1}^m \int_{\Omega_e} \nabla N_i^e \cdot \left[\gamma \nabla N_j^e \varphi_j + \frac{\partial}{\partial t}(\varepsilon \nabla N_j^e \varphi_j) \right] d\Omega \right\} = \\ & - \sum_{e=1}^{m_1} \oint_{\Gamma_1} N_i^e \left[\gamma E_n + \frac{\partial}{\partial t}(\varepsilon E_n) \right] d\Gamma + \\ & \sum_{e=1}^{m_2} \oint_{\Gamma_2} N_i^e \left[\gamma \psi + \frac{\partial}{\partial t}(\varepsilon \psi) \right] d\Gamma + \\ & \sum_{e=1}^{m_3} \oint_{\Gamma_{12}} N_i^e \left[\gamma \frac{\partial \varphi_1}{\partial n_{1e}} + \frac{\partial}{\partial t} \left(\varepsilon \frac{\partial \varphi_1}{\partial n_{1e}} \right) + \gamma \frac{\partial \varphi_2}{\partial n_{2e}} + \right. \\ & \quad \left. \frac{\partial}{\partial t} \left(\varepsilon \frac{\partial \varphi_2}{\partial n_{2e}} \right) \right] d\Gamma \end{aligned} \quad (11)$$

where $\mathbf{n}_{1e}$ and $\mathbf{n}_{2e}$ are the unit normal vector of the boundary $\partial\Omega_e$ in the Γ_1 side and Γ_2 side, respectively. Thus, $\mathbf{n}_{2e} = -\mathbf{n}_{1e}$.

By using (4), the curve integral along the dielectric interface in (11) is equal to 0. If the medium is isotropic, linear, and homogeneous, the conductivity and permittivity of the medium are constant. Thus, (11) can be written as:

$$\begin{aligned} & \sum_{j=1}^n \left(\sum_{e=1}^m \int_{\Omega_e} \gamma \nabla N_i^e \cdot \nabla N_j^e d\Omega \right) \varphi_j + \\ & \frac{\partial}{\partial t} \left[\sum_{j=1}^n \left(\sum_{e=1}^m \int_{\Omega_e} \varepsilon \nabla N_i^e \cdot \nabla N_j^e d\Omega \right) \varphi_j \right] = \\ & - \sum_{e=1}^{m_1} \int_{\Gamma_{1e}} \gamma N_i^e E_n d\Gamma - \frac{\partial}{\partial t} \left(\sum_{e=1}^{m_1} \int_{\Gamma_{1e}} \varepsilon N_i^e E_n d\Gamma \right) + \\ & \sum_{e=1}^{m_2} \int_{\Gamma_{2e}} \gamma N_i^e \psi d\Gamma + \frac{\partial}{\partial t} \left(\sum_{e=1}^{m_2} \int_{\Gamma_{2e}} \varepsilon N_i^e \psi d\Gamma \right) \end{aligned} \quad (12)$$

Equation (12) can be written as the matrix form,

$$\mathbf{K}_\gamma \Phi + \frac{\partial}{\partial t} (\mathbf{K}_\varepsilon \Phi) = \mathbf{f}_\gamma + \frac{\partial}{\partial t} (\mathbf{f}_\varepsilon) \quad (13)$$

where $\mathbf{K}_\gamma$ and $\mathbf{K}_\varepsilon$ are the stiff matrix of the conductivity and the permittivity respectively, $\mathbf{f}_\gamma$ and $\mathbf{f}_\varepsilon$ are vectors of the boundary integrals in the right hand of (12) related to the conductivity and the permittivity, respectively, and Φ is the column vector of the nodal potential. Expressions for $\mathbf{K}_\gamma$, $\mathbf{K}_\varepsilon$, $\mathbf{f}_\gamma$ and $\mathbf{f}_\varepsilon$ can be given as:

$$\mathbf{K}_\gamma = \sum_{j=1}^n \sum_{e=1}^m \int_{\Omega_e} \gamma \nabla N_i^e \cdot \nabla N_j^e d\Omega \quad (14)$$

$$\mathbf{K}_\varepsilon = \sum_{j=1}^n \sum_{e=1}^m \int_{\Omega_e} \varepsilon \nabla N_i^e \cdot \nabla N_j^e d\Omega \quad (15)$$

$$\mathbf{f}_\gamma = - \sum_{e=1}^{m_1} \int_{\Gamma_{1e}} \gamma N_i^e E_n d\Gamma + \sum_{e=1}^{m_2} \int_{\Gamma_{2e}} \gamma N_i^e \psi d\Gamma \quad (16)$$

$$\mathbf{f}_\varepsilon = - \sum_{e=1}^{m_1} \int_{\Gamma_{1e}} \varepsilon N_i^e E_n d\Gamma + \sum_{e=1}^{m_2} \int_{\Gamma_{2e}} \varepsilon N_i^e \psi d\Gamma \quad (17)$$

The time derivative of function Ψ can be discretized as time difference form [14]:

$$\Psi^{(n+1)} - \Psi^{(n)} = \Delta t [\beta \dot{\Psi}^{(n+1)} + (1 - \beta) \dot{\Psi}^{(n)}] \quad (18)$$

where $\dot{\Psi}$ stands for the time derivative of function Ψ , superscript (n) and $(n+1)$ stand for time step n and $(n+1)$, Δt is the time step, β is a coefficient, $\beta = 0, 0.5, 1$ stand for the forward Euler method, the backward Euler method, and the Crank—Nicolson method, respectively. When $\beta \gg 0.5$, the algorithm is unconditionally stable.

Thus, by using (18), matrix (13) can be expressed as:

$$\begin{aligned} & (\mathbf{K}_\varepsilon + \Delta t \beta \mathbf{K}_\gamma) \Phi^{(n+1)} = (\mathbf{K}_\varepsilon - \Delta t (1 - \beta) \mathbf{K}_\gamma) \Phi^{(n)} + \\ & \Delta t \beta \mathbf{f}_\gamma^{(n+1)} + \Delta t (1 - \beta) \mathbf{f}_\gamma^{(n)} + \mathbf{f}_\varepsilon^{(n+1)} - \mathbf{f}_\varepsilon^{(n)} \end{aligned} \quad (19)$$

where the-vectors of right-hand items of (18) are expressed as:

$$\mathbf{f}_\gamma^{(n+1)} = - \sum_{e=1}^{m_1} \int_{\Gamma_{1e}} \gamma N_i^e E_n^{(n+1)} d\Gamma +$$

$$\sum_{e=1}^{m_2} \int_{\Gamma_{2e}} \gamma N_i^e \psi^{(n+1)} d\Gamma \quad (20)$$

$$\mathbf{f}_\gamma^{(n)} = - \sum_{e=1}^{m_1} \int_{\Gamma_{1e}} \gamma N_i^e E_n^{(n)} d\Gamma + \sum_{e=1}^{m_2} \int_{\Gamma_{2e}} \gamma N_i^e \psi^{(n)} d\Gamma \quad (21)$$

$$\begin{aligned} \mathbf{f}_\varepsilon^{(n+1)} = & - \sum_{e=1}^{m_1} \int_{\Gamma_{1e}} \varepsilon N_i^e E_n^{(n+1)} d\Gamma + \\ & \sum_{e=1}^{m_2} \int_{\Gamma_{2e}} \varepsilon N_i^e \psi^{(n+1)} d\Gamma \end{aligned} \quad (22)$$

$$\mathbf{f}_\varepsilon^{(n)} = - \sum_{e=1}^{m_1} \int_{\Gamma_{1e}} \varepsilon N_i^e E_n^{(n)} d\Gamma + \sum_{e=1}^{m_2} \int_{\Gamma_{2e}} \varepsilon N_i^e \psi^{(n)} d\Gamma \quad (23)$$

Equation (19) can be expressed as a matrix equation:

$$\mathbf{S} \Phi^{(n+1)} = \mathbf{T} \Phi^{(n)} + \mathbf{G}^{(n)} + \mathbf{F}^{(n+1)} \quad (24)$$

where matrix $\mathbf{S}$ and $\mathbf{T}$ can be expressed as:

$$\mathbf{S} = \mathbf{K}_\varepsilon + \Delta t \beta \mathbf{K}_\gamma \quad (25)$$

$$\mathbf{T} = \mathbf{K}_\varepsilon - \Delta t (1 - \beta) \mathbf{K}_\gamma \quad (26)$$

In addition, the elements of vector $\mathbf{G}$ and $\mathbf{F}$ can be given as:

$$g_i^{(n)} = \begin{cases} 0 & \text{in } \Omega \\ - \sum_{e=1}^{m_2} \int_{\Gamma_{2e}} [\varepsilon - \Delta t (1 - \beta) \gamma] N_i^e \psi^{(n)} d\Gamma & \text{on } \Gamma_2 \\ \sum_{e=1}^{m_1} \int_{\Gamma_{1e}} [\varepsilon - \Delta t (1 - \beta) \gamma] N_i^e E_n^{(n)} d\Gamma & \text{on } \Gamma_1 \end{cases} \quad (27)$$

$$f_i^{(n+1)} = \begin{cases} 0 & \text{in } \Omega \\ \sum_{e=1}^{m_2} \int_{\Gamma_{2e}} [\varepsilon + \Delta t \beta \gamma] N_i^e \psi^{(n+1)} d\Gamma & \text{on } \Gamma_2 \\ - \sum_{e=1}^{m_1} \int_{\Gamma_{1e}} [\varepsilon + \Delta t \beta \gamma] N_i^e E_n^{(n+1)} d\Gamma & \text{on } \Gamma_1 \end{cases} \quad (28)$$

III. TRANSIENT CONSTRAINED ELECTRIC FIELD EQUATION ON THE BOUNDARY UNDER THE EQS FIELD

A. Transient Constrained Electric Field Equation on the Boundary

After discretization in space and time, the finite element equation of the initial value and boundary value problem under the EQS condition can be obtained, and the matrix form is shown in (24). When numbering the nodes, the nodes that are in Ω but not on the Γ_1 are numbered first. Then, nodes on the Γ_1 with given nodal potentials are numbered in sequence. As a result, (24) can be represented with a partitioned matrix, as shown in (29),

$$\begin{bmatrix} \mathbf{S}_{11} & \mathbf{S}_{21}^T \\ \mathbf{S}_{21} & \mathbf{S}_{22} \end{bmatrix} \begin{bmatrix} \Phi^{(n+1)} \\ \mathbf{U}^{(n+1)} \end{bmatrix} = \begin{bmatrix} \mathbf{T}_{11} & \mathbf{T}_{21}^T \\ \mathbf{T}_{21} & \mathbf{T}_{22} \end{bmatrix} \begin{bmatrix} \Phi^{(n)} \\ \mathbf{U}^{(n)} \end{bmatrix} + \begin{bmatrix} \mathbf{G}_2^{(n)} \\ \mathbf{G}_1^{(n)} \end{bmatrix} + \begin{bmatrix} \mathbf{F}_2^{(n+1)} \\ \mathbf{F}_1^{(n+1)} \end{bmatrix} \quad (29)$$

where Φ is a n_1 order column vector of the nodal potential which needs to be solved, U is a $(n - n_1)$ order column vector of the nodal potential with known potential values on the Dirichlet boundary, S_{11} , S_{21} , and S_{22} are $n_1 \times n_1$ order, $(n - n_1) \times n_1$ order, $(n - n_1) \times (n - n_1)$ order partitioned stiffness matrixes of S , respectively, T_{11} , T_{21} , and T_{22} are $n_1 \times n_1$ order, $(n - n_1) \times n_1$ order, $(n - n_1) \times (n - n_1)$ order partitioned stiffness matrixes of T , respectively, G_2 is a n_1 order partitioned column vector of G whose element value is known while G_1 is a $(n - n_1)$ order partitioned column vector of G related to the normal component of the electric field intensity on the Dirichlet boundary, F_2 is a n_1 order partitioned column vector of F with a known element value, and F_1 is a $(n - n_1)$ order partitioned column vector of F related to the normal component of the electric field intensity on the Dirichlet boundary, and needs to be solved.

Equation (29) can be decomposed into the following two matrix equations,

$$\Phi^{(n+1)} = S_{11}^{-1}(T_{11}\Phi^{(n)} + T_{21}^T U^{(n)} + G_2^{(n)} + F_2^{(n+1)} - S_{21}^T U^{(n+1)}) \quad (30)$$

$$F_1^{(n+1)} = S_{21}\Phi^{(n+1)} + S_{22}U^{(n+1)} - T_{21}\Phi^{(n)} - T_{22}U^{(n)} - G_1^{(n)} \quad (31)$$

Equation (30) is the time-domain finite element equation to solve the transient nodal potential in the EQS field. Generally, the Dirichlet boundary is related to the surface of the electrode, whose transient electric field intensity only has the normal component. Even if there are tangential components on the Dirichlet boundary, (31) also can fully reflect the constraint relationship between the normal component of the transient electric field intensity and the transient potential. Assume that the normal component of the transient electric field intensity on the Newmann boundary is known, thus, (31) is named as the transient constrained electric field equation on the boundary (TCEFEB) for calculating the normal component of the transient electric field intensity on the Dirichlet boundary in the EQS field.

Element values of column vector F_1 can be obtained from (28) and can be expressed as:

$$\sum_{e=1}^{m_1} \int_{\Gamma_{1e}} [\varepsilon + \Delta t \beta \gamma] N_i^e E_n^{(n+1)} d\Gamma = -f_i^{(n+1)} \quad (i = n_1 + 1, \dots, n) \quad (32)$$

Based on the base function, the normal component of the transient electric field intensity on the Dirichlet boundary can be given as:

$$\tilde{E}_n^{(n)} = \sum_{j=n_1+1}^n N_j E_{nj}^{(n)} \quad (33)$$

where $N_j (j = n_1 + 1, \dots, n)$ is the nodal basis function at node j on the Dirichlet boundary and $E_{nj}^{(n)}$ is the normal component of the nodal electric field intensity at node j on the Dirichlet boundary at time step n .

Inputting (33) into (32), element values of column vector F_1 represented by the normal component of the nodal electric

field intensity on the Dirichlet boundary can be expressed as:

$$\sum_{j=n_1+1}^n \left[\sum_{e=1}^{m_1} \int_{\partial\Omega_{1e}} (\varepsilon + \Delta t \beta \gamma) N_i^e N_j^e d\Gamma \right] E_{nj}^{(n+1)} = -f_i^{(n+1)} \quad (34)$$

Equation (34) can be expressed in the form of a matrix equation as:

$$K E_n^{(n+1)} = -F_1^{(n+1)} \quad (35)$$

where K is a stiff matrix related to the electric field intensity on the Dirichlet boundary, and E_n is the column vector of the normal component of the electric field intensity on the Dirichlet boundary. Element values of stiff matrix K are calculated by:

$$k_{i,j} = \sum_{e=1}^{m_1} \int_{\partial\Omega_{1e}} (\varepsilon + \Delta t \beta \gamma) N_i^e N_j^e d\Gamma \quad (i, j = n_1 + 1, \dots, n) \quad (36)$$

Similarly, the column vector $G_1^{(n)}$ is satisfied with the following matrix equation:

$$G_1^{(n)} = H E_n^{(n)} \quad (37)$$

where elements value of stiff matrix H are calculated by:

$$h_{i,j} = \sum_{e=1}^{m_1} \int_{\partial\Omega_{1e}} [\varepsilon - \Delta t(1 - \beta)\gamma] N_i^e N_j^e d\Gamma \quad (i, j = n_1 + 1, \dots, n) \quad (38)$$

Taking (35) and (37) into (31), the TCEFEB for calculating the normal component of the electric field intensity on the Dirichlet boundary under the EQS field can be expressed as.

$$E_n^{(n+1)} = -K^{-1}(S_{21}\Phi^{(n+1)} + S_{22}U^{(n+1)} - T_{21}\Phi^{(n)} - T_{22}U^{(n)} - H E_n^{(n)}) \quad (39)$$

After calculating the nodal potential in each time step, the normal component of the electric field intensity on the Dirichlet boundary can be calculated step by step by (39).

B. Transient Charge Density on Dielectric Interface

For the composite insulation structure, the interface charge density can be calculated by (5). Thus, the normal component of the electric field intensity on the dielectric interface should be further calculated.

In order to solve the normal component of the electric field intensity on the dielectric interface by TCEFEB, the following special treatment is arranged by decomposing filed domain Ω into several subdomains Ω_i according to the following procedure.

$$\Omega = \bigcup_{i=1}^{m_{sd}} \Omega_i \quad (40)$$

$$\Omega_i \cap \Omega_j = \Gamma_{ij} (\Gamma_{ij} \neq \emptyset, j = 1, 2, \dots, m_{sd}) \quad (41)$$

where m_{sd} denotes the number of subdomains, Ω_i and Ω_j denote subdomain i and subdomain j with only one kind of medium, Γ_{ij} denotes interface between subdomain Ω_i and subdomain Ω_j .

In subdomain Ω_i , the nodal potential at every time step is obtained by (29), thus all dielectric interfaces between subdomain Ω_i and other subdomains and the Dirichlet boundary of subdomain Ω_i can be regarded as the equivalent Dirichlet boundary,

$$\Gamma'_{i1} = \Gamma_{i1} + \sum_{j=1, j \neq i} \Gamma_{ij} \quad (42)$$

where Γ_{i1} is the Dirichlet boundary of subdomain Ω_i , and Γ'_{i1} is the equivalent Dirichlet boundary of subdomain Ω_i .

Thus, the normal component of the transient electric field intensity at each time step on the equivalent Dirichlet boundary of subdomain Ω_i can be calculated by (28). Repeating the above procedures for all subdomains, normal components of the transient electric field intensities at each time step on the Dirichlet boundary and the interface of different mediums of domain Ω can be solved. As for choosing a reference direction for the normal component of the electric field intensity, the outer normal vector at the boundary of subdomain Ω_i is selected.

We consider that the directions of the outer normal vectors on each side of interface Γ_{ij} are opposite. The transient electric charge density on the interface of different mediums can then be calculated by:

$$\sigma_s = \varepsilon_i E_{in} + \varepsilon_j E_{jn} \quad (43)$$

where σ_s is the transient interface charge density, ε_i and ε_j are permittivities in subdomain Ω_i and subdomain Ω_j respectively, E_{in} and E_{jn} are the normal component of the transient electric field on the interface Γ_{ij} at different sides of subdomain Ω_i and subdomain Ω_j respectively.

In summary, the block diagram of the algorithm flow for calculating the normal component of the transient electric field on the Dirichlet boundary and dielectric interface, as well as transient charge density on the interface is illustrated in Fig. 2.

Because of the unconditionally numerical stability and second-order calculation accuracy of the Crank-Nicolson difference method, the Crank-Nicolson difference method is used to discretize the equations by numerical code in this paper. It should be pointed out that the Crank-Nicolson difference scheme will originate numerical oscillation at the beginning of the calculation or when the state variable is abruptly changed. In order to eliminate the numerical oscillation caused by the Crank-Nicolson difference method, the backward Euler method is used in the calculation of nodal potential as the first time step [37]. At this step, the time step is changed from Δt to $\Delta t/2$, after using the backward Euler method twice in a row, and the nodal potential $\Phi^{(1)}$ at step 1 will be obtained. Then, nodal potentials at each subsequent time step are calculated according to the Crank-Nicolson difference method.

IV. NUMERICAL EXAMPLE

A. Calculation Model and the Analytical Solution

A coaxial model with double medium layers is used to verify the validity of the proposed method, as shown in Fig. 3. The outer conductor ($r = R_2$) is grounded, and the step voltage $u_s(t) = U_s u(t)$ kV is applied to the inner conductor

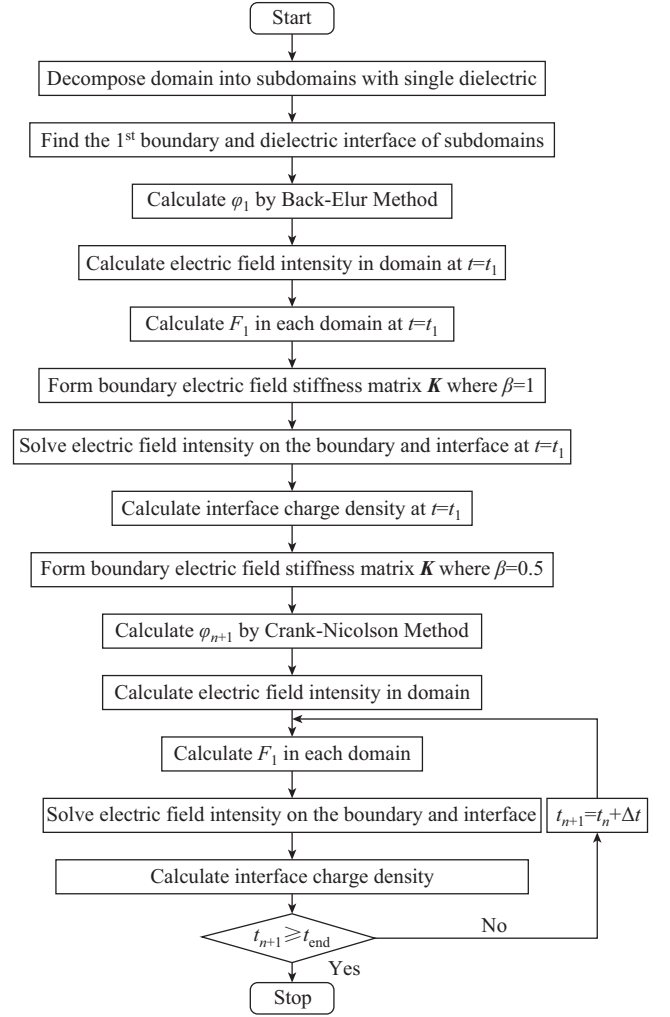


Fig. 2. Flow diagram of calculating interface charge density under the EQS field by TCEFEB.

($r = R_0$), where, $u(t)$ is a unite step function and U_s is the amplitude of the applied voltage. The outer radius of the inner conductor (R_0), medium layer 1 (R_1), and medium layer 2 (R_2) are 0.02 m, 0.03 m, and 0.05 m, respectively. The relative permittivity and the conductivity of medium layer 1 are $\varepsilon_{r1} = 4.106$ and $\gamma_1 = 2.55 \times 10^{-17}$ S/m, while for layer 2, $\varepsilon_{r2} = 3.177$ and $\gamma_2 = 6.0 \times 10^{-16}$ S/m.

The electric field intensity of the model has only the axial component. The transient electric field intensity can be obtained analytically and can be expressed as:

$$E_1 = \frac{1}{\rho} \left[\frac{\gamma_2 U_s}{\gamma_1 \ln \frac{R_2}{R_1} + \gamma_2 \ln \frac{R_1}{R_0}} (1 - e^{-\frac{t}{\tau_c}}) + \frac{\varepsilon_2 U_s}{\varepsilon_1 \ln \frac{R_2}{R_1} + \varepsilon_2 \ln \frac{R_1}{R_0}} e^{-\frac{t}{\tau_c}} \right] \quad (44)$$

$$E_2 = \frac{1}{\rho} \left[\frac{\gamma_1 U_s}{\gamma_1 \ln \frac{R_2}{R_1} + \gamma_2 \ln \frac{R_1}{R_0}} (1 - e^{-\frac{t}{\tau_c}}) + \frac{\varepsilon_1 U_s}{\varepsilon_1 \ln \frac{R_2}{R_1} + \varepsilon_2 \ln \frac{R_1}{R_0}} e^{-\frac{t}{\tau_c}} \right] \quad (45)$$

where E_1 and E_2 are the module of electric field intensity in dielectric 1 and 2, respectively, ρ is the distance from field

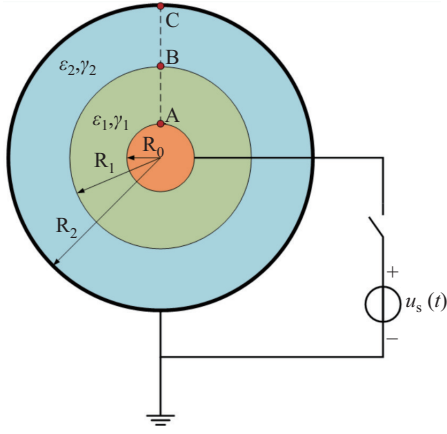


Fig. 3. A coaxial model with double medium layers.

point to inner conductor center, τ_e stands for charge relaxation time and can be given as:

$$\tau_e = \frac{\varepsilon_1 \ln \frac{R_2}{R_1} + \varepsilon_2 \ln \frac{R_1}{R_0}}{\gamma_1 \ln \frac{R_2}{R_1} + \gamma_2 \ln \frac{R_1}{R_0}} \quad (46)$$

The analytical solution of the transient charge density on the dielectric interface is:

$$\sigma = \varepsilon_2 E_2 - \varepsilon_1 E_1 = \frac{\varepsilon_2 \gamma_1 - \varepsilon_1 \gamma_2}{\gamma_1 \ln \frac{R_2}{R_1} + \gamma_2 \ln \frac{R_1}{R_0}} U_s (1 - e^{-\frac{t}{\tau_e}}) \quad (47)$$

B. Accuracy Comparison of Electric Field Intensity

To represent changes of axial electric field intensity with time, a line segment along the axial direction is selected, whose coordinates at start point A and stop point B are (0 m, 0.02 m) and (0 m, 0.05 m) respectively, and point C (0 m, 0.03 m) is the intersection of the line segment and dielectric interface, as shown in Fig. 3. The field domain is meshed by using 2-D linear triangular elements with 837 nodes and 1550 elements. In the calculation, the amplitude of the voltage U_s is set as 10 kV.

The analytical and numerical results of the transient electric field intensity on the Dirichlet boundary and dielectric interface are shown in Fig. 4. The solid lines of different colors represent the analytical results of the electric field intensity along the line segment at different time steps. The points with different colors are the numerical results of the electric field intensities on the Dirichlet boundary and the dielectric interface at different time steps calculated by the proposed method. In this example, the charge relaxation time is about 60 s calculated by (45), thus 60 s is selected as the time interval. As shown in Fig. 4, the numerical results of the electric field strength on the Dirichlet boundary and dielectric interface calculated by the proposed method in this paper are in good agreement with the analytical results.

Generally, there are three traditional methods to calculate electric field intensity on the boundary. In order to explain these methods, the relationships of nodes and elements on the Dirichlet boundary are shown in Fig. 5.

Method 1 is taking the element electric field intensity at the center of the element, which is collinear with the boundary to approximate the electric field intensity on the boundary, and

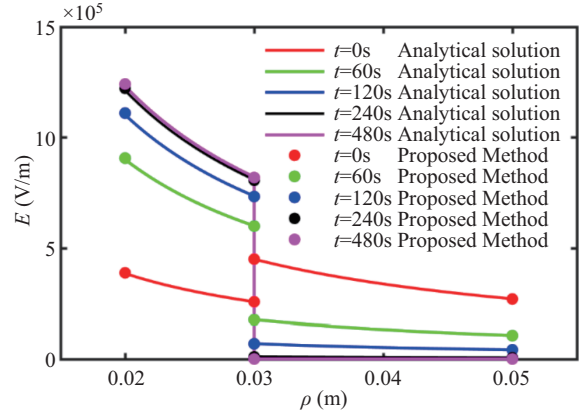


Fig. 4. Comparison of analytical and numerical results of the electric field intensity on the Dirichlet boundary and the interface at different time.

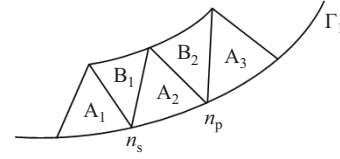


Fig. 5. Schematic of nodes and elements on the Dirichlet boundary.

taking the arithmetic mean value of the element electric field intensity on the two sides of the node, which is calculated as the nodal electric field intensity on the boundary. For instance, the element electric field of element A_2 is taken as the electric field intensity of curve segment $n_s n_p$ on the boundary, and the arithmetic mean value of element electric field intensity of element A_2 and A_3 is regarded as the nodal electric field intensity of node n_p [35]. Method 2 is taking the element electric field intensity at the center of the element, which has only one vertex on the boundary to approximate the nodal electric field intensity on the boundary. For example, the element electric field intensity of element B_2 is considered as the nodal electric field intensity of node n_p [35]. Method 3 is taking the element area-weighted average value of element electric field intensities associated with the node to approximate the nodal electric field intensity on the boundary. For instance, for node n_p on the boundary, the nodal electric field intensity is regarded as the elemental area-weighted average value of elemental electric field intensities of A_2 , B_2 , and A_3 [38].

The formula of the element area-weighted average value of the element electric field intensity associated with the node i is:

$$E_{k,i} = \frac{\sum_{j=1}^{N_{ne}} E_{k,ij}^e S_{ij}^e}{\sum_{j=1}^{N_{ne}} S_{ij}^e} \quad (48)$$

where $E_{k,i}$ denotes the component of the nodal field intensity at the direction k of node i , N_{ne} denotes the number of elements associated with node i , $E_{k,ij}^e$ denotes the element electric field intensity component in the direction k of element j associated with node i , and S_{ij}^e denotes the element area of element j associated with node i .

Numerical results of the transient electric field intensity of

start point A on the line segment by the above three methods, the proposed method in this paper, and its analytical results are shown in Fig. 6.

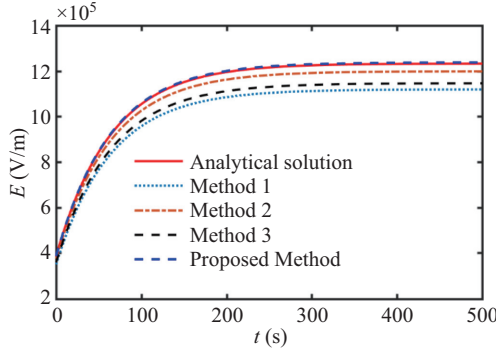


Fig. 6. Comparison of numerical results of the transient electric field intensity calculated by four different methods and the analytical result at start point A on the Dirichlet boundary.

As depicted in Fig. 6, numerical results of the transient electric field intensity at point A on the Dirichlet boundary calculated by the proposed method in this paper are very well in agreement with the analytical result, while the numerical results obtained by the other three methods have some certain errors compared with the analytical results. The accuracy of the proposed method is higher than the other three methods in the calculation of the electric field intensity on the Dirichlet boundary.

In order to compare the accuracy of the transient electric field intensities on the Dirichlet boundary and the dielectric interface calculated by the three traditional methods and the proposed method, the field domain of the coaxial model is meshed by 2-D linear triangular elements with three different mesh densities. The numbers of nodes and elements of three mesh discretization densities are shown in Table I.

TABLE I
THREE DIFFERENT MESH DISCRETIZATION

Mesh kinds	Mesh 1	Mesh 2	Mesh 3
Num. of element	1550	5140	19474
Num. of node	837	2682	9959

The transient electric field intensities on the Dirichlet boundary and the dielectric interface by the four methods

under three mesh discretization densities are calculated. Their average relative errors are also calculated and illustrated in Table II. In the calculation, the amplitude of the voltage U_s is also set as 10 kV. To show discontinuity of the electric field intensity on the dielectric interface, let $\rho = 0.03_-$ which stands for the inner side of the dielectric interface of dielectric layer 1, and $\rho = 0.03_+$ which stands for the outer side of the dielectric interface of dielectric layer 2.

The average relative error of the proposed method and other three traditional methods for the electric field intensity is calculated by:

$$\delta_E = \frac{1}{N_t} \sum_{n=1}^{N_t} \left(\frac{1}{M} \sum_{i=1}^M \frac{||\mathbf{E}_{ic}^{(n)}|| - ||\mathbf{E}_i^{(n)}||}{||\mathbf{E}_i^{(n)}||} \times 100\% \right) \quad (49)$$

where δ_E is the average relative error of the electric field intensity of all nodes on the boundary at all time steps, $\mathbf{E}_{ic}^{(n)}$ is the numerical results of the electric field intensity of node i at time step n , $\mathbf{E}_i^{(n)}$ is the analytical result of the electric field intensity of node i at time step n , M is the total number of nodes on the boundary, and N_t is the total number of time steps.

As depicted in Table II, the accuracy of the transient electric field intensities on the Dirichlet boundary and dielectric interface calculated by the proposed method in this paper is much higher than the other three traditional methods in the same mesh discretization. Under any of the same mesh discretizations, the accuracy of the proposed method is above an order of magnitude higher than the other three traditional methods. With the increase of mesh density, although the accuracy of the other three methods will improve, the accuracy of the proposed method in this paper will improve even more. Even if the mesh density increases by 10 times (compared to Mesh 3 and Mesh 1), the accuracy of the three traditional methods under mesh 3 is also lower than the accuracy of the proposed method under mesh 1. Therefore, more accurate numerical results will be obtained even if mesh density is lower for the proposed method compared with other three traditional methods.

C. Accuracy Comparison of the Interface Charge Density

The numerical results of the transient charge density of point C by the above three types of different methods, the proposed method, and its analytical results are all shown in Fig. 7. The

TABLE II
AVERAGE RELATIVE ERROR OF DIFFERENT METHODS FOR ELECTRIC FIELD INTENSITY

Mesh	Average relative error of nodal electric potential (%)	Location (m)	Average relative error of nodal electric field intensity (%)			
			Method 1 (%)	Method 2 (%)	Method 3 (%)	Proposed method (%)
Mesh 1	0.0509	$\rho = 0.02$	9.2165	2.9844	7.0838	0.2743
		$\rho = 0.03_-$	6.3667	2.5223	5.1328	0.3876
		$\rho = 0.03_+$	7.0923	4.2196	6.0931	0.8939
		$\rho = 0.05$	4.2065	2.2598	3.5785	0.0927
		$\rho = 0.02$	6.0347	3.6319	5.1998	0.2893
Mesh 2	0.0532	$\rho = 0.03_-$	3.6865	1.8076	3.0774	0.0747
		$\rho = 0.03_+$	3.7596	1.8465	3.1096	0.2085
		$\rho = 0.05$	2.2180	1.0919	1.8489	0.0259
		$\rho = 0.02$	2.7385	1.2162	2.2239	0.0237
		$\rho = 0.03_-$	1.8902	0.9057	1.5665	0.0081
Mesh 3	0.0040	$\rho = 0.03_+$	1.8577	0.8157	1.5073	0.0275
		$\rho = 0.05$	1.1282	0.5383	0.9332	0.0056

red solid line denotes the analytical result of the transient charge density, while the dot lines with other colors denote numerical results calculated by the four methods.

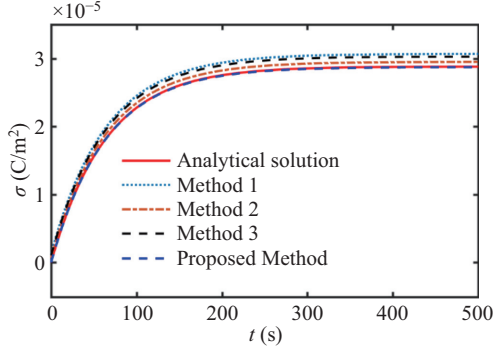


Fig. 7. Comparison of numerical results of the transient charge density calculated by the four methods and the analytical result at point C on the dielectric interface under Mesh 1.

As shown in Fig. 7, the numerical result of the transient charge density at point C on the dielectric interface calculated by the proposed method agrees very well with the analytical results, while the numerical results obtained by the other three methods have some certain errors compared with the analytical result. Numerical results calculated by the other three methods are all greater than the analytical results. It is obvious that the accuracy of the proposed method is higher than the other three methods in calculation charge density on the dielectric interface.

When $t = 0$, the electric field distributes according to the electrostatic field, and the charge density is 0 on the analytical dielectric interface. However, the numerical result of the charge density on the dielectric interface is not equal to 0 because of the numerical error. When $t = 0$, numerical results of the charge density at point C on the dielectric interface by the three traditional methods are $9.9749 \times 10^{-7} \text{ C/m}^2$, $9.4054 \times 10^{-7} \text{ C/m}^2$, and $9.6985 \times 10^{-7} \text{ C/m}^2$ respectively. While the numerical results calculated by the proposed method in this paper under the same condition is $1.0625 \times 10^{-13} \text{ C/m}^2$. When $t > 0$, the values of the transient charge density ranges from 10^{-7} C/m^2 to 10^{-5} C/m^2 for the analytical results. As a consequence, the three traditional methods cannot distinguish situations without interface charge density in the electrostatic field and situations with accumulated interface charge density. But, the numerical results of the charge density on the dielectric interface calculated by the proposed method in this paper is about 10^{-13} C/m^2 when $t = 0$, which is five orders of magnitude smaller than the transient interface charge density when $t > 0$. Thus, the proposed method in this paper can distinguish situations free of the interface charge in the

electrostatic field condition.

Similar to the analysis in section IV-B, average relative errors of the charge density on the dielectric interface calculated by the three methods and the proposed method under three mesh discretization densities are shown in Table III. In the calculation of average relative error, results at $t = 0$ are ignored, because the analytical result of the charge density is 0 when $t = 0$. The average relative error of the nodal charge density is:

$$\delta_\sigma = \frac{1}{N_t - 1} \sum_{n=2}^{N_t} \left(\frac{1}{M} \sum_{i=1}^M \frac{||\sigma_{ic}^{(n)}| - |\sigma_i^{(n)}||}{|\sigma_i^{(n)}|} \times 100\% \right) \quad (50)$$

where δ_σ is the average relative error of the charge density of all nodes on the dielectric interface at all time steps, $\sigma_{ic}^{(n)}$ is the numerical results of the charge density of node i at time step n , $\sigma_i^{(n)}$ is the analytical result of the charge density of node i at time step n , M is the total number of nodes on the dielectric interface, and N_t is the total number of time steps.

Compared with the other three traditional methods, the average relative errors of the proposed method are more than an order of magnitude lower in the same mesh density, and close to the average relative error of nodal potential, as shown in Table III. With the increase of mesh density, average relative errors of the three traditional methods will decrease, but remain on the same order of magnitude. When mesh density increases from Mesh 1 to Mesh 3, the average relative errors of the three traditional methods decrease by 3.44 times, 3.12 times, and 3.39 times, respectively, however, the average relative error of the proposed method decreases by 46.38 times. Even if the mesh density increases by 10 times (compared to Mesh 3 and Mesh 1), the average relative errors of the three traditional methods under Mesh 3 are also higher than the accuracy of the proposed method in this paper under Mesh 1. Therefore, compared to the traditional methods, more accurate numerical results will be obtained even if mesh density is lower. It can be concluded that the accuracy of the transient charge density on the dielectric interface calculated by the proposed method in this paper is much higher than the other three traditional methods in the same mesh density.

D. Discussions

In traditional FEM, the penalty function method or imposed boundary condition is used to deal with the Dirichlet boundary condition. However, important information about electric field intensity on the boundary is not considered. The TCEFEB is derived from Galerkin's finite element formulation with some modifications under the EQS field, as illustrated in sections 2.2 and 2.3. First, the normal component of the electric field intensity on the boundary is introduced when considering physical meaning. $-\partial\varphi/\partial n$ stands for the normal component

TABLE III
AVERAGE RELATIVE ERROR OF DIFFERENT METHODS FOR INTERFACE CHARGE DENSITY

Mesh	Average relative error of nodal electric potential (%)	Average relative error of nodal interface charge density (%)			
		Method 1 (%)	Method 2 (%)	Method 3 (%)	Proposed method (%)
Mesh 1	0.0509	9.2945	3.9942	7.5762	0.3803
Mesh 2	0.0532	5.2962	2.5953	4.4141	0.0731
Mesh 3	0.0040	2.7021	1.2783	2.2323	0.0082

of the electric field intensity E_n , thus, $\partial\varphi/\partial n$ is represented by $-E_n$ in the curve integral along the Dirichlet boundary in the right hand of (10). Secondly, with the proper numbering of nodes, matrix decomposition can be performed on (24), and partitioned matrix (24) can be derived. As a result, the electric potentials in the domain and on the Neumann boundary and the normal component of the electric field intensity on the Dirichlet boundary can be calculated by (30) and (31) step by step, respectively. Thirdly, the penalty function method or method of imposed boundary condition used for dealing with the Dirichlet boundary condition is not required. In summary, with some proper modifications to the existing FEMs' code, the normal component of the electric field intensity on the boundary can be calculated.

In terms of an algorithm, the other three traditional methods are approximated in the node electric field intensity on the boundary by the average value of element electric field intensity associated with the node on the boundary. The differential error is unavoidable when the elemental electric field intensity is calculated by taking the gradient of the nodal potential. As a result, the calculation accuracy of the elemental electric field intensity will be much lower than that of the nodal potential. In addition, errors will accumulate and increase when calculating the nodal electric field intensity by using the average value of the element electric field intensity associated with the node.

However, in the proposed method of this paper, the normal component of the transient electric field intensities on the Dirichlet boundary and the interface is directly solved by (31). Thus, taking the gradient of electric potential is not required, which can eliminate calculation error due to the numerical differentiation. At the same time, errors caused by average approximation can also be eliminated. As a result, the accuracy of the electric field intensity is much higher than the existing three traditional methods.

The transient nodal potential is solved by (30), and the solution is definitely determined by the potential on the Dirichlet boundary and the normal derivative of the potential on the Neumann boundary. These boundary conditions and initial values are precisely given. The calculation accuracy of nodal potential is very high, because the error is originated from FEM whose error is very small. The transient electric field intensity on the Dirichlet boundary is calculated by (31), and the solution is determined by the given potential on the Dirichlet boundary and the obtained transient nodal potential. As a consequence, the numerical error of the nodal potential will influence the calculation of the electric field intensity on the boundary. Thus, the numerical error of the transient electric field intensity on the Dirichlet boundary is a little higher than that of the nodal potential. Similarly, numerical errors of the transient electric field intensity and the transient charge density on the interface are also a little higher than that of the nodal potential. Even so, the calculation accuracy of this method is still much higher than other traditional methods for the transient electric field intensities on the Dirichlet boundary and the interface as well as the transient interface charge density.

As for advantages in the engineering application of the TCEFEB, first, it is useful to optimize the design of the electrode shape of high voltage equipment. The electrode

is corresponding to the Dirichlet boundary in the electric field analysis, and there is only the normal component in the electric field intensity on the electrode, which can be directly and accurately calculated by the proposed TCEFEB. Secondly, it is useful to optimize the insulation design of high voltage equipment with combined insulation structures. Calculation of interface charge density is essential for the design of combined insulation structures, because the interface charge will modulate the distribution of the electric field. By decomposing the filed domain into several subdomains with a single medium, the normal component of the transient electric field intensity on the dielectric interface can be calculated by the TCEFEB. Then, the interface charge density can be calculated with high accuracy by (5).

V. CONCLUSION

A time-domain finite element method for calculation of the transient electric field intensity on the dielectric interface under the EQS field is proposed in this paper. The governing equation represented by the nodal potential is discretized by nodal FEM in the space and finite difference method in time. The following conclusions can be made:

The transient constrained electric field equation on the boundary (TCEFEB) with different difference formats is derived, which can be used for directly calculating the normal component of the transient electric field intensity on the Dirichlet boundary. By decomposing the filed domain into several subdomains with a single medium, the normal component of the transient electric field intensity on the dielectric interface can be calculated by the TCEFEB. Then the transient charge density on the dielectric interface can also be calculated.

For calculating the normal component of the transient electric field intensity on the Dirichlet boundary and the dielectric interface, the accuracy of the proposed method in this paper is close to the accuracy of the nodal potential and much higher than the existing traditional FEMs. For calculating the transient charge density on the dielectric interface, the average relative error of the three traditional methods are 9.2945%, 3.9942%, and 7.5762%, respectively, under proper mesh density with 837 nodes and 1550 elements, and that of the proposed method is 0.3803%. When increasing mesh density with 9959 nodes and 19474 elements, the average relative error of the three traditional methods are 2.7021%, 1.2783%, and 2.2323%, respectively, and that of the proposed method is 0.0082%. Even if the mesh density is increased by about 10 times, the calculation accuracy of the traditional FEMs cannot reach the proposed method in this paper. Compared with the traditional methods, the calculation accuracy of the proposed method in this paper is far more significantly improved by increasing the mesh density.

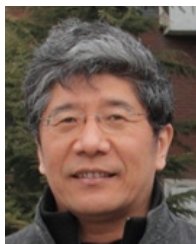
The TCEFEB is derived from the Galerkin's finite element formulation under the EQS field. With some proper modifications to the existing FEMs' code, the transient electric field intensities on the Dirichlet boundary and interface as well as the transient charge density with high accuracy will be calculated. It's convenient to apply to the optimized design of electrode shape and insulation structure of the electric equipment with combined insulation structures.

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